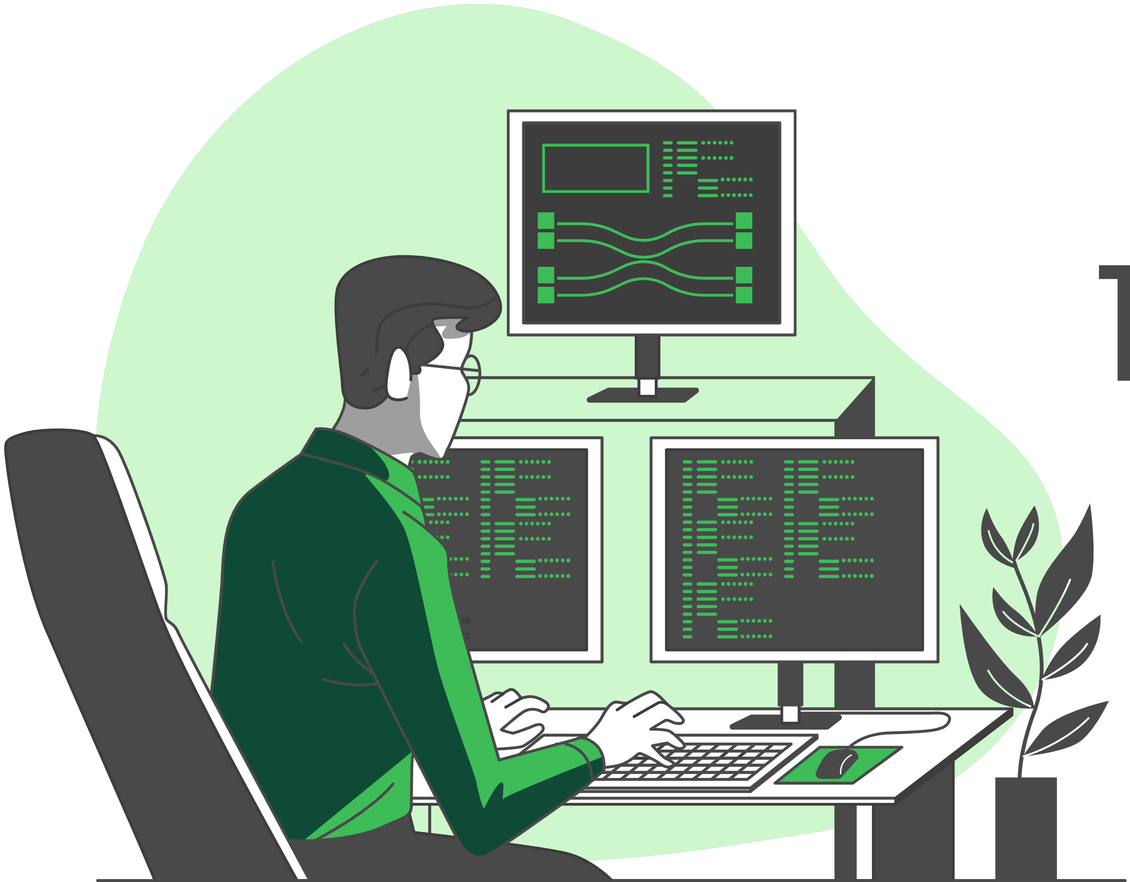

Topological Ordering

CS 251 - Data Structures and Algorithms

Note:
Slides complement the discussion in class

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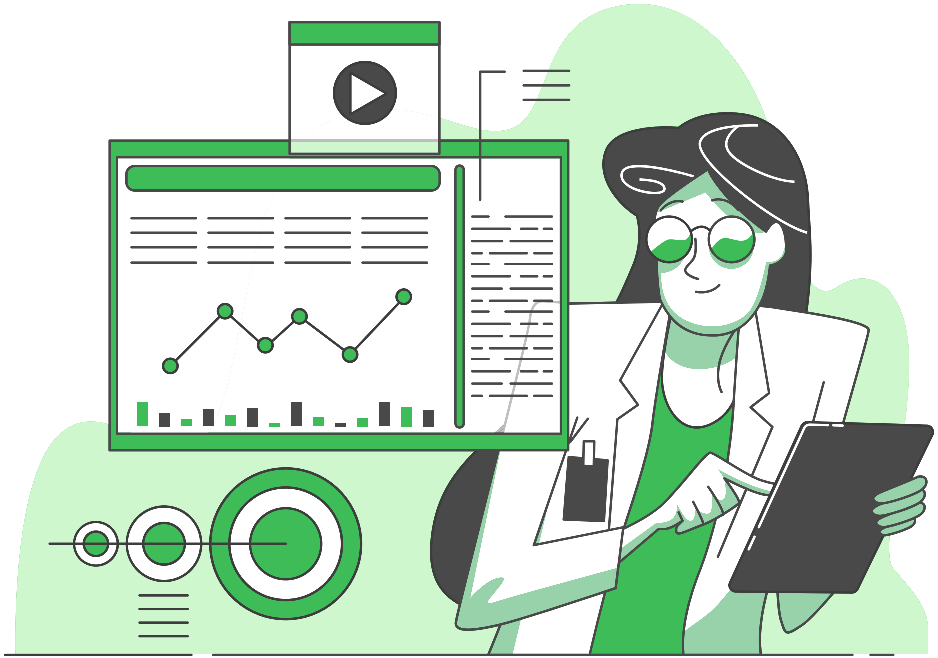


01

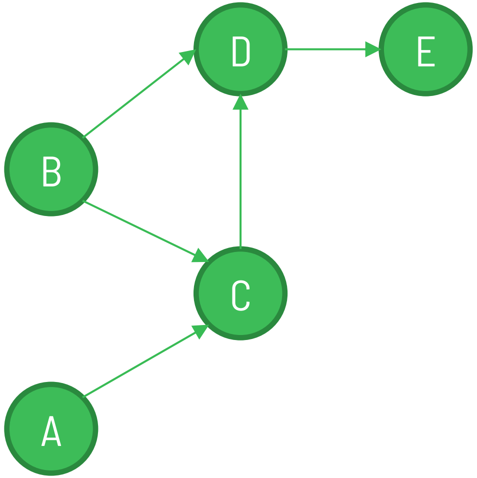
Topological Ordering

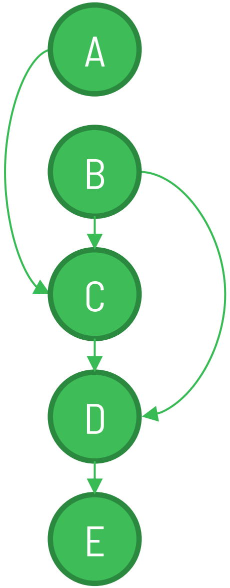
When graphs have no cycles

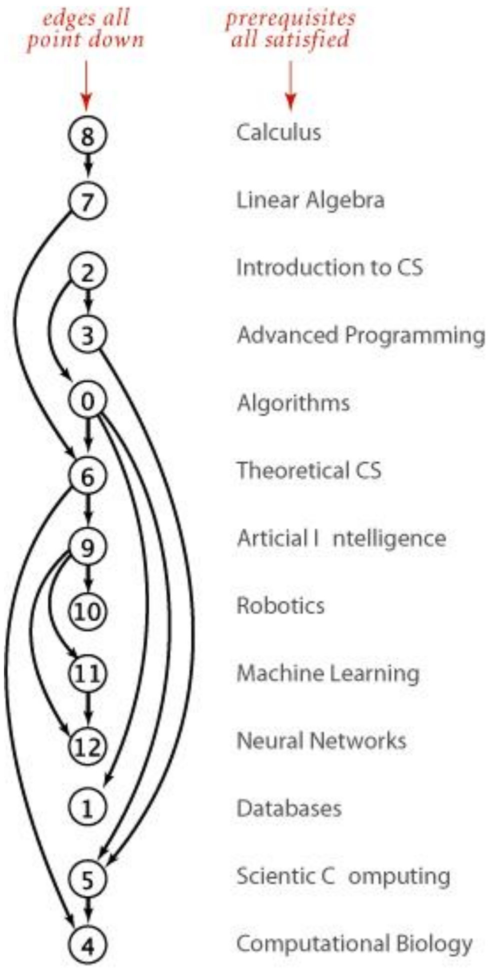
DAGs and Topological Ordering

A **Directed Acyclic Graph** (DAG) is a digraph that has no directed cycles.

A **Topological Ordering** of a digraph is a numbering $v_1, \ldots, v_n$ of the vertices of the graph such that for every edge (v_i, v_j), we have $i < j$.







Topological sort

Algorithms, 4th Edition. Sedgewick and Wayne, Chapter 4

Topological Sort Algorithm

algorithm TopologicalSort($G(V,E)$)
  let H be a copy of G
  $n \leftarrow 0$
  let $T\colon v \in V \rightarrow \mathbb{Z}_{\geq 0}$

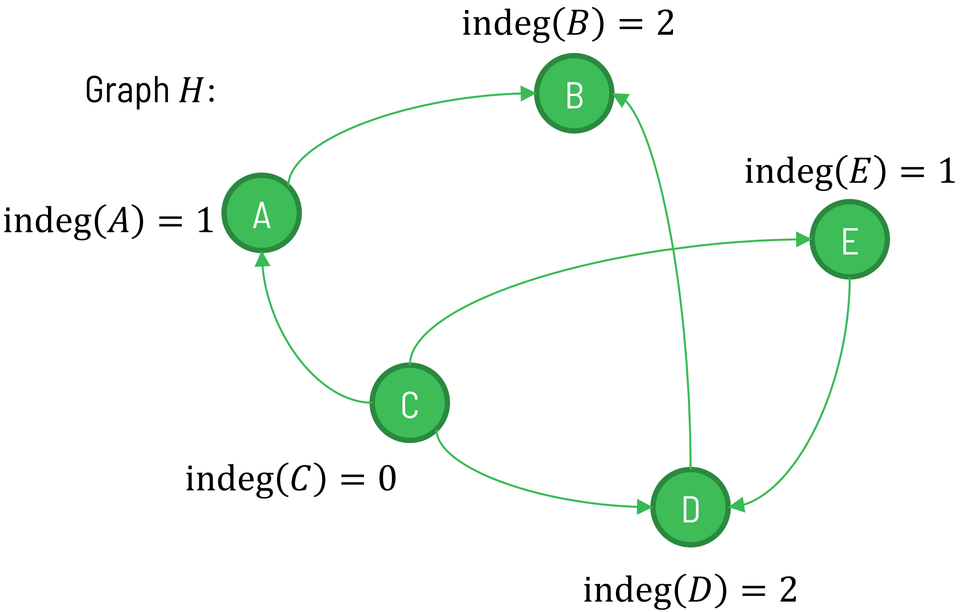
  while H is not empty **do**
    pick $v \in H.V$ s.t. indeg$(v) = 0$
    $T[v] \leftarrow n$
    $n \leftarrow n + 1$
    remove v from H
  end while

  return T
end algorithm

Run DFS using a Stack to keep track of the current path (and determine if there are cycles)

Example:

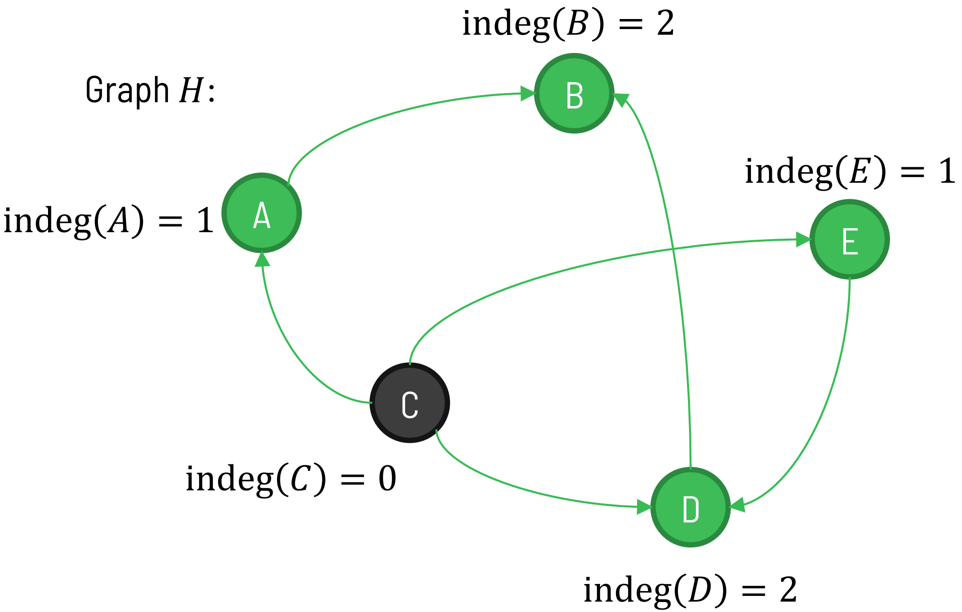
Graph H:



$n = 0$

$T = \{\}$

Example:



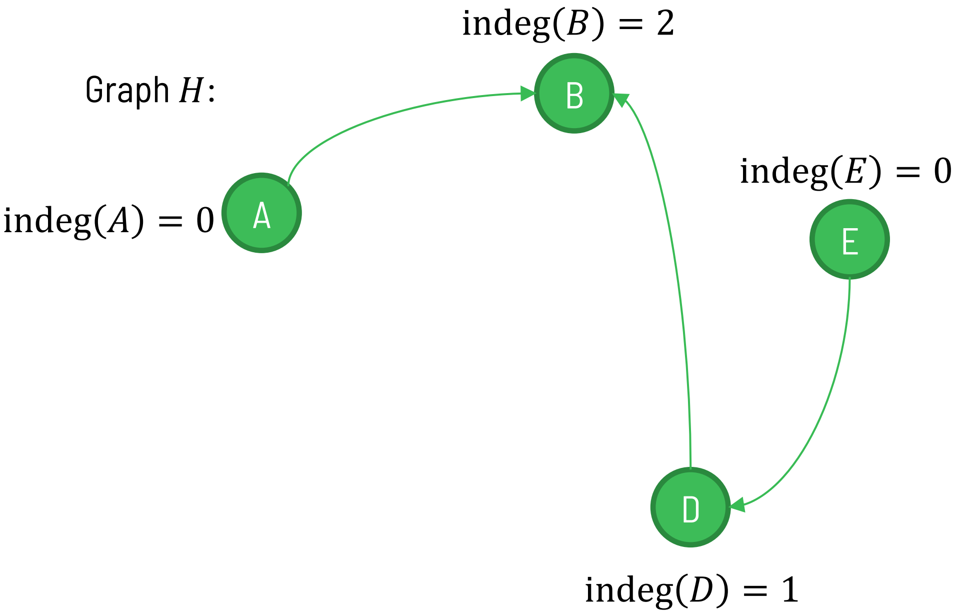
$\text{indeg}(B) = 2$
Graph H:
B
$\text{indeg}(E) = 1$
$\text{indeg}(A) = 1$
A
E
C
$\text{indeg}(C) = 0$
D
$\text{indeg}(D) = 2$

$n = 0$

$T = \{\}$

Example:

Graph H:

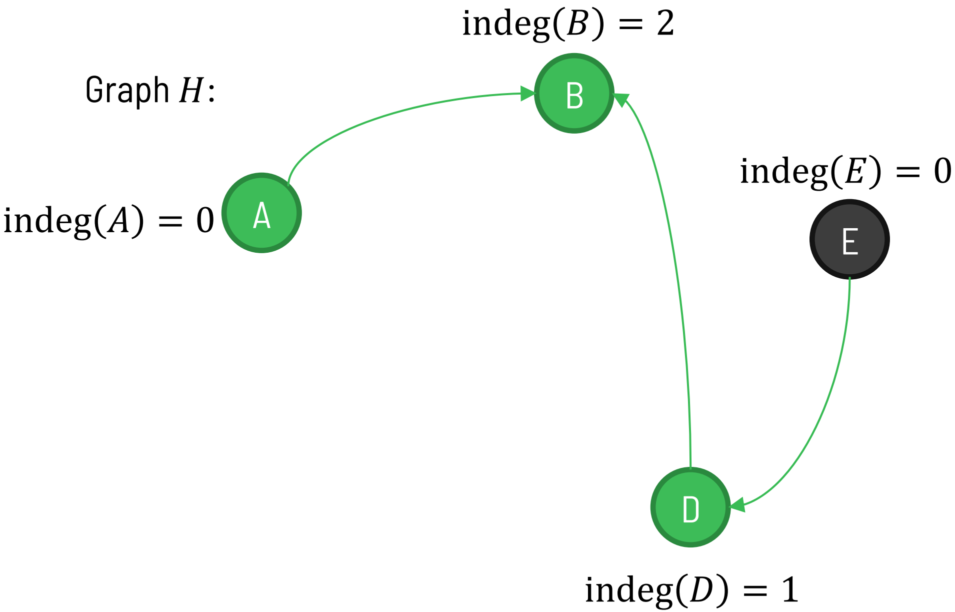


$n = 1$

$T = \{D: 0\}$

Example:

Graph H:

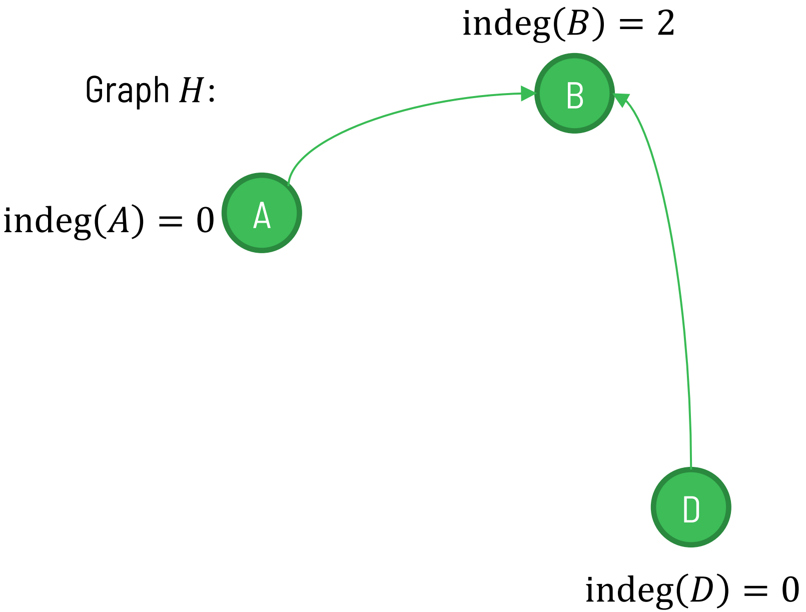


$n = 1$

$T = \{D: 0\}$

Example:

Graph H:

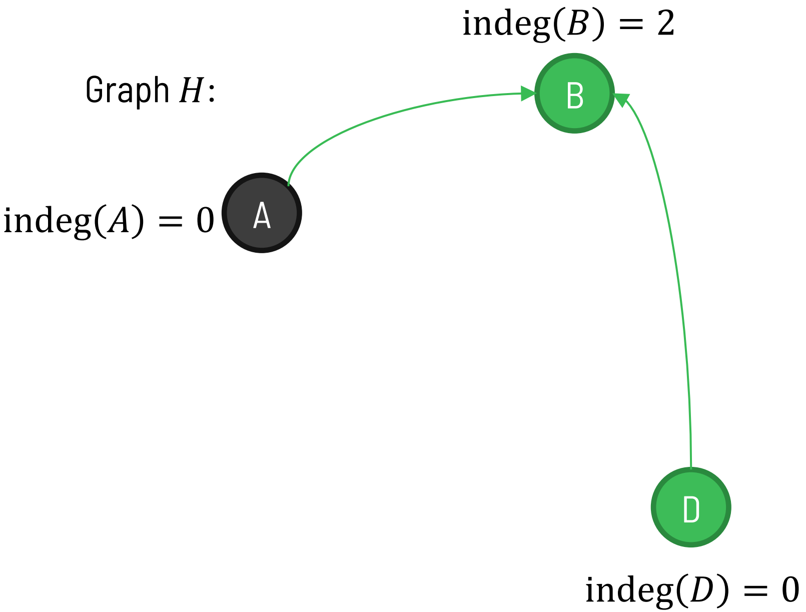


$n = 2$

$T = \{D: 0, E: 1\}$

Example:

Graph H:



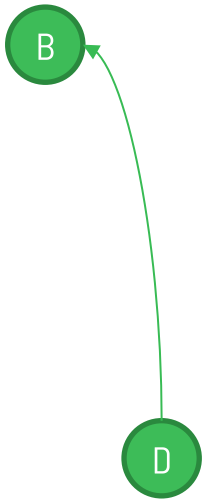
$n = 2$

$T = \{D: 0, E: 1\}$

Example:

Graph H:

$\text{indeg}(B) = 1$



$\text{indeg}(D) = 0$

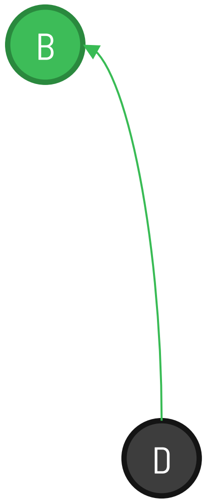
$n = 3$

$T = \{D: 0, E: 1, A: 2\}$

Example:

Graph H:

$\text{indeg}(B) = 1$



$\text{indeg}(D) = 0$

$n = 3$

$T = \{D: 0, E: 1, A: 2\}$

Example:

$\mathrm{indeg}(B) = 0$

Graph H:

$n = 4$

$T = \{D: 0, E: 1, A: 2, D: 3\}$

Example:

$\text{indeg}(B) = 0$

Graph H:

$n = 4$

$T = \{D: 0, E: 1, A: 2, D: 3\}$

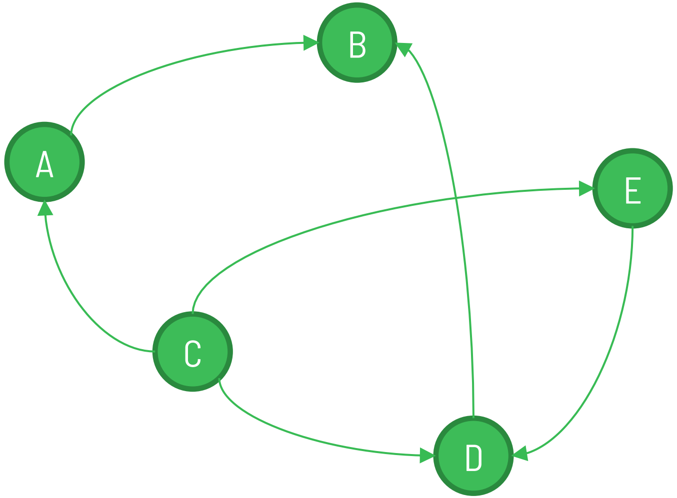
Example:

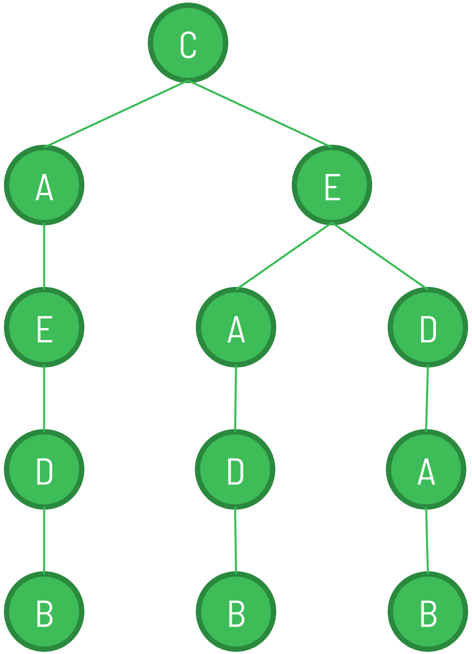
Graph H:

$$n = 5$$

$$T = \{D: 0, E: 1, A: 2, D: 3, B: 4\}$$

Tree of Topological Orderings





Ordo Sine Circuitu

Do you have any questions?

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